Tools of the Trade:

ARITHMETIC vs. LOGARITHMIC LINE CHARTS

Graphic presentation of rates or ratios is commonplace in health statistics reports. Line charts are particularly useful in showing rates over time or for a series of values, such as age-specific death rates. When displaying data on a line chart, an arithmetic or linear scale is almost always used on the horizontal and vertical axes of the chart. On an arithmetic scale, equal distances represent equal amounts. Thus, the distance from a value of 1 to 2 is the same as the distance from 2 to 3, 3 to 4, and so on. A simple example of a line chart using an arithmetic scale on both axes is shown below. It is easy for both the presenter and reader to use and understand a line chart plotted on arithmetic scales. Basic mathematical principles are involved.

However, an alternative scale that can be used with a line chart in many instances can actually be more useful in comparing rates. A logarithmic or semi-logarithmic line chart has a logarithmic scale on the y (vertical) axis and an arithmetic scale on the x (horizontal) axis. These charts are useful for comparing relative (percentage) changes, rather than absolute amounts of change, for a set of values. On a logarithmic scale, equal distances represent equal ratios. For example, the distance from 1 to 2 is the same as that from 2 to 4, 4 to 8, 8 to 16, etc. - at each interval, a ratio of 1 to 2. An example of a logarithmic line chart with a logarithmic scale on the y axis is also show below.

A data series plotted on an arithmetic scale that increases or decreases at a constant rate will appear as a curving line, ascending at an increasing angle for a growing series of values and descending at a decreasing angle for a diminishing set of values. A data series plotted on a logarithmic line chart using a logarithmic scale on the y axis will resemble a straight line, slanting up or down according to the data values. Constant increases or decreases will appear as a straight line. The examples below help show these differences.

The line chart at the left below presents data plotted on standard arithmetic scales. It is useful for reading the absolute amount by which each value changes; however, it does not show relative change. It simply shows that one data series (Data A) increased by a greater amount than the other (Data B). The example on the right below presents the same two data sets plotted on a logarithmic line chart using a logarithmic scale on the y axis. This chart is useful for comparing the rates at which each value changes. Absolute values can be read from the scale at the side of the chart, but not compared visually. This chart shows that Data B increased at a greater rate than Data A and reveals that the rate of increase became progressively larger in Data B and progressively smaller in Data A. Much more information about the differences between these two data series is obtained from the logarithmic line chart.

ARITHMETIC LINE CHART

LOGARITHMIC LINE CHART
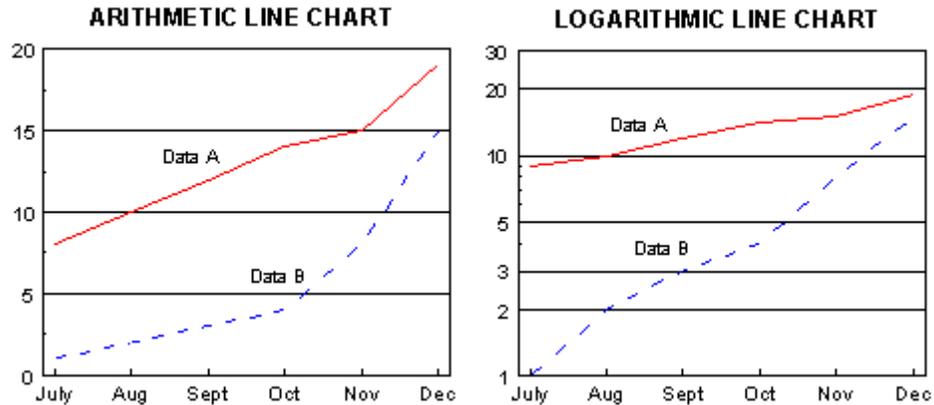
The two line charts below demonstrate the difference between the two scales when plotting the same health data, i.e., age-specific cancer incidence rates among Pennsylvania residents in 1988. The first line chart uses the more common arithmetic scale on both axes. It is useful for reading the absolute amount by which each value changes. From this chart, it seems that there is little or no difference in the rates for males and females until about ages 65-69 when male rates become much higher and stay much higher. However, the logarithmic line chart, using a logarithmic scale on the y axis, shows the percentage changes in the same rates. In this chart, the female rates for the younger age groups seem somewhat higher compared to the male rates and the percentage differences in the rates for the older age groups are not as evident as in the arithmetic line chart.

In terms of comparison, the logarithmic line chart provides a more complete description of the relationship between these two series of rates. In other words, the logarithmic chart points to a possible significant difference between the rates at the younger age groups, whereas in the arithmetic line chart the difference at the younger age groups is lost in the plotting of the higher absolute values for the older age groups.

After these examples, you may begin to wonder why logarithmic line charts are not used more often. The virtues of logarithmic line charts can be important; however, they are gained at a cost. Since most people expect a level twice as high to be twice as great, they can be badly misled by these types of charts if they do not understand the type of comparison being shown. Another important reason is that logarithmic line charts cannot be used to show zeroes or negative values. Also, they are not so easy to construct as arithmetic charts; however, computer software today has automated most graphic production. Unfortunately, these types of charts look overly technical and discourage readers from trying to understand them. For these reasons, they are not recommended for widespread use or inclusion in statistical reports with the general public as the targeted audience. They are, however, excellent for specialized purposes and with technical audiences.

Age-Specific Cancer Incidence Rates,
Pennsylvania Residents, 1988

ARITHMETIC SCALE
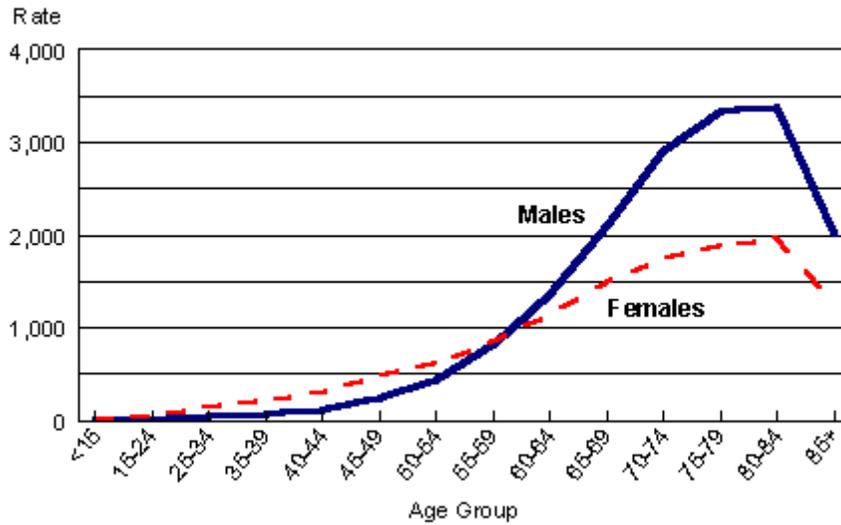
LOGARITHMIC SCALE
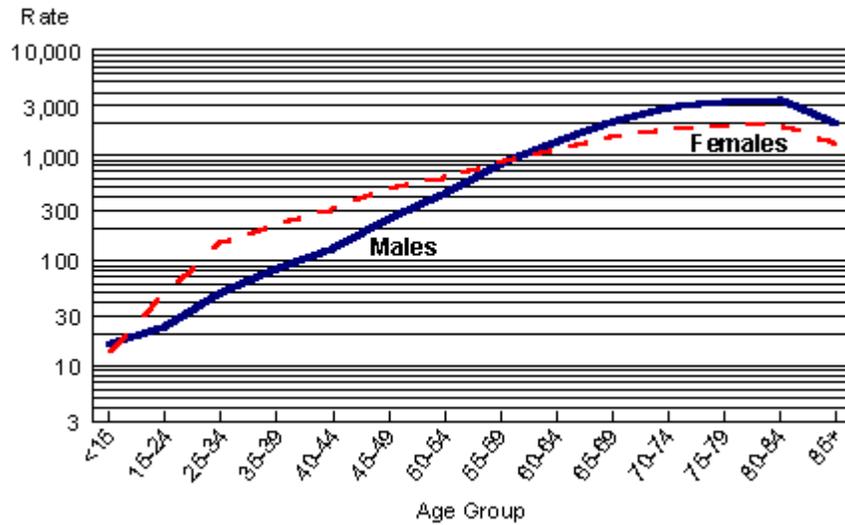
NOTE: Rates are per 100,000 population for each specified age group.